

Queen Mary, University of London
MAS317 Linear Algebra II, 2007
Coursework 1

This coursework should be handed in by 1 pm on MONDAY October 1st in the ?box on ? floor in the mathematics building.

Late coursework will not be accepted. Write your name and student number at the top and clip all the sheets together.

1. **[30 marks]** State the definition of the *transpose* of a matrix A , and what is meant by saying that A is *congruent to* B for A and B $n \times n$ matrices over some field.

Prove the following results about $n \times n$ matrices. You may use the results in (1.8), and should state which part you are using at different points in your proofs.

- (i) If A is congruent to B and B is congruent to C then A is congruent to C .
- (ii) If A is congruent to B then B is congruent to A .
- (iii) If A is congruent to a diagonal matrix then A is symmetric.

2. **[30 marks]** Let $A = \begin{pmatrix} 1 & 3 & -2 \\ 3 & 0 & 3 \\ -2 & 3 & 7 \end{pmatrix}$.

Use elementary row and column operations, as in Method A, to find a diagonal matrix D over $\mathbb{Q}$ and an invertible matrix P over $\mathbb{Q}$ such that $D = P^T A P$.

Note: *The answers for D and P are not unique. One useful check: starting from fixed A , all the possible D 's have the same numbers of positive terms, negative terms and zeros on the diagonal. We will prove this later.*

3. **[40 marks]** Let $B = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix}$.

Work out how to repeat Question 2 with the matrix B instead of A throughout. You will need to start by finding a way of coping with the zeros on the diagonal of B .